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ABSTRACT

This paper discusses a model to optimize cash management in school districts. A brief discussion of the cash flow pattern of school districts is followed by an analysis of the constraints faced by the school districts in their investment planning process. A linear programming model used to optimize net interest earnings on investments is developed and discussed. The paper concludes that the use of this cash management technique can result in millions of additional revenue dollars for school districts every year. (Author/LD)

A MODEL FOR DETERMINING SCHOOL DISTRICT CASH FLOW NEEDS

BY

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Introduction

Financial planning is an important function in school district business management because of the differences in the timing of receipts and expenditures. School districts receive most of their revenues through property taxes and state aid payments. Both of these payments accrue to the school district in large lump sums periodically throughout the fiscal year. The property taxes, which account for 40% to 60% of the total district revenues, is usually received by the school district once or twice a year. Thus, these annual or semi-annual payments are quite large.

State aid payments are usually the second largest source of school district revenue. The timing and amounts of state aid payments vary from state to state. However, these payments are also quite large. Together, the property tax receipts and state aid payments comprise the majority of school district receipts, typically accounting for between 80% to 90% of the total revenue of the district. Fortunately, both the timing and amounts of these receipts are known with a high degree of vertainty by the school district financial planners. This enables the business staff to anticipate the receipt of these funds which aids in the development of a cash flow schedule.

Besides being able to anticipate revenues with a high degree of certainty, expenditures may also be anticipated. School district expenditures are comprised primarily of salary and fringe benfit expenses. The typical school district budget allots approximately 80% of the expenditures for these purposes. Once the number of district employees is set, and the salary schedules are negotiated, these expenditures may be anticipated with a high degree of certainty. Except for the Summer months,

the expenditure pattern of most school districts is quite stable. Other major expenses, such as debt service payments and utility bills may also be anticipated through a review of previous year's expenditure patterns. June is usually the highest expenditure month because of social security and tax payments to the Federal Government as well as end of the year payments.

Figure 1. presents a simplified revenue and expenditure pattern of a hypothetical school district. October and January revenues are large because of property tax receipts while the November and March revenues are large because of state aid payments. The expenditure for most months is stable at $200,000 per month, with a high in June and low expenses in the Summer months. Because of the differences in the timing of revenues and expenditures, the school districts face periods of surplus revenues over expenditures, denoted by "S". January through March are periods of surplus cash. From April through October, however, the school district has more expenditures than revenues. These periods of cash deficit are denoted by "D".

The primary task of the financial planners of the school district is to determine what investments, if any, should be made. If investments are made, three questions have to be answered: when should investments be made?, what amounts should be invested?, and how long should the cash be invested? If it is necessary for the school district to borrow money until its revenues arrives, the same three questions about timing, amount, and maturity need to be answered.

This financial planning task is usually approached in the following manner. The net cash flow is determined by subtracting the monthly expenditures from revenues. A positive (negative) net flow indicates a

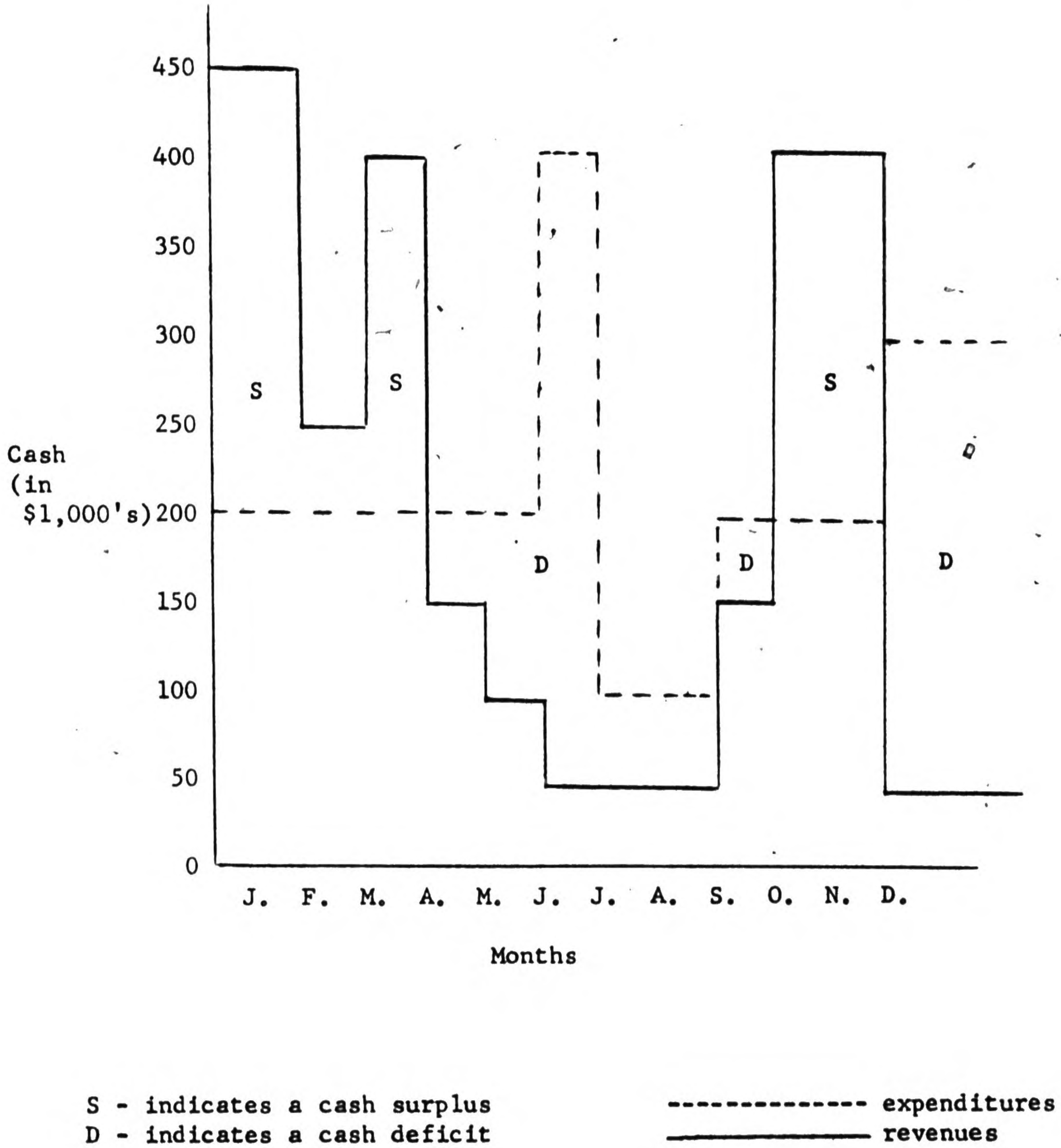

Figure 1.

The Cash Budget and Cash Flow Pattern of the Sample District

cash surplus (deficit). Table 1. presents the monthly cash flows in tabular form. The month of January has a $200,000 cash surplus. This cash should be invested in interest-bearing assets such as a Certificate of Deposit or Repurchase Agreement. Since the month of April has a cash deficit of $50,000, an investment of $50,000 should be made from January to April. Another $100,000 could be invested from January to May to cover that deficit, with the remaining $50,000 from January invested until June. Because revenues equal expenditures, this process will be followed until all surplus cash is invested to mature during the months of cash deficit situations.

The astute reader will note that this process will work until the month of July. In July, the school district needs $50,000 to meet expenses, but all available revenue is used up. This deficit situation continues until October when the property tax arrives. By that time, the district will have accumulated a deficit of $150,000. Clearly, the district must borrow $150,000 in anticipation of future revenues. In October, sufficient revenues will arrive to allow for the repayment of the anticipation note, as well as to allow for the investment of an additional $50,000. Thus, the investment process starts again.

This investment process is typically followed by most school districts. Unfortunately, if the school district planners determined the investments in such a fashion, it would most likely lose revenue through lost interest. For example, suppose the financial planners were concerned with the period from January to June only, eliminating the necessity to include borrowing in its determinations. During this six-month period, there are a number of opportunities to invest surplus cash. The important question for the financial planners to answer is: what investments should be made in order

TABLE 1

The Cash Flow Pattern of the Sample District

Period	0	1	2	3	4	5	6	7	8	9	10	11	12
Month	Jan. 1	Feb. 1	Mar. 1	Apr. 1	May 1	June 1	July 1	Aug. 1	Sept. 1	Oct. 1	Nov. 1	Dec. 1	Dec. 3
Inflow	400	250	400	150	100	50	50	50	150	400	400	50	0
Outflow	200	200	200	200	200	400	100	100	200	200	200	300	0
Netflow	+200	+50	+200	-50	-100	-350	-50	-50	-50	+200	+200	-250	0

All revenues are received by the district on the first day of the month.

All expenditures occur on the first day of the month.

The beginning cash balance of the district on the first day of the year is $50,000.

to maximize interest earnings while making sure that all bills get paid? This is a classical constrained optimization question.

The Use of Linear Programming in School District Financial Management

Linear Programming is a mathematical technique used primarily to solve constrained optimization problems. Constrained optimization problems are concerned with the attainment of specified objectives given limiting constraints. Smythe and Johnson list the following steps in formulating a linear programming model:

A. Recognition of the problem,

B. Formulation of the mathematical model:
   1. identification of the decision variables,
   2. choice of a measure of effectiveness,
   3. symbolic representation of the objective function,
   4. identfication of constraints,
   5. algebraic representation of constraints.

C. Estimation of the parameters of the model.

These steps will be followed in the development of the linear programming formulation of school district financial management. Only the investment applications to financial management will be covered in this report. A linear program that covers both borrowing and investment is very complex, and beyond the scope of this report.

Using only the first six month period of the example school district's cash flow pattern given in Table 1., it is evident that there are ample opportunities for investment of surplus funds. The financial planners' task is to determine when to make investments, how much cash to invest, and when to have the investments mature. For simplicity of notation, suppose we call January "month 1", February "month 2", etc., with June being "month 6". Any investments which could be made will be denoted as NP_{tk}. NP stands for

"notes purchased". The subscript t represents the month when the note is purchased. The subscript k denotes the number of months that will pass until the note matures. Thus, the notation NP15 would represent an investment note purchased in the first month that will mature at the beginning of the sixth month.

In the planning period in question, there are fifteen possible investments: NP11 to NP15, NP21 to NP24, NP31 to NP33, NP41, NP42, and NP51. Which of these investments should be made, and in what amounts? <u>If all investments earned the same annual rate of interest, it would make no difference as to either the timing or amounts of the investments!</u> The total interest earning would be the same regardless of the investment made. However, it is a financial fact that there is a term structure of interest rates. The longer the duration of the investment, the higher the annual rate of interest will be offered in the financial markets.

For this example, an investment of 1 to 2 month's duration will earn 7% annually or .07/12=.0058/month interest. An investment of 3 to 4 months duration will earn 9% annually or .09/12=.0075/month, and an investment of 5 to 6 months duration will earn 11% annually or .11/12=.0092 monthly. A call to the local bank will obtain the current term structure of interest rates.

Using the above term structure, NP15 would earn (5 months x .0092/month) = .046NP15 interest. Thus, if NP15 were equal to $250,000 invested in month 1, maturing at the beginning of month 6, it woulf earn (5 x .0092 x $250,000) = $11,500 interest. Likewise, if NP32 was $150,000, it would earn (2 x .0058 x $150,000) = $1,740 interest.

Given the term structure of interest rates, there is only <u>one</u> set of investments which will maximize the total interest earnings possible for

the school district. It is next to impossible to determine the optimal set of investments using hand methods. To accomplish this task, linear programming must be used. This discussion was used to accomplish the first step in formulating a linear programming problem: the recognition of the problem.

The second step is the formulation of the mathematical model. The identification of the decision variables requires the development of a system of notation to represent the variables used in this problem. NPtk has been established as the notation for the investment possibilities. Likewise, IDtk will be used to represent the interest due on investments NPtk. Thus, the interest due on investment NP24 is denoted as ID24. The monthly revenue inflows will be denoted as It, so the revenue that arrives in month 4 = I4 = $150,000. The monthly expenditure that flows out of the district is denoted as Ot. The monthly expenditure in month 6 = O6 = $400,000. The last notation necessary represents the cash balance left at the end of the month. The cash balance will be represented by Ct. Because there is a beginning cash balance at the start of the fiscal year in month 1, this will be denoted as C0. Thus, the cash balance left at the end of the second month = C2. In summary, the variables used in the linear program are:

NPtk = investment notes purchased in month t maturing in k months,

IDtk = the interest due on investment NPtk,

It = the inflow of revenue in month t,

Ot = the outflow of expenditure in month t,

Ct = the cash balance left at the end of month t.

The next step in the formulation of the linear programming problem is the selection of a measure of effectiveness. Since the objective is to maximize total interest earnings, the measure of effectiveness is dollars.

Given the measure of effectiveness, the objective function is to maximize the dollars earned in interest on investment. Since there are 15 possible investments, there are 15 possible interest earnings. If Z represents the total interest earning, then the objective function is stated formally as:

$$\text{Maximize } Z = \sum IDtk$$

or $$Z = ID11 + ID12 + ID13 + ID14 + ID15 + ID21 + ID22 + ID23 + ID24 + ID31 + ID32 + ID33 + ID41 + ID42 + ID51.$$

With the objective function stated, the next steps are to identify and mathematically state the relevant constraints. For this simplified problem, there is only one constraint: the cash balance constraint. At the beginning of this section, it was noted that the purpose of the linear program was to maximize interest earnings while making sure that all bills are paid. Each period (t), the following sequence of events takes place: the inflow of revenue It is added to the cash balance remaining from the previous period Ct-1. Investments from previous periods may mature, so the principal NPtk plus interest due IDtk is added to the available cash. Bills are paid, so the monthly expenditure Ot is deducted from the available cash. If there is surplus cash, investments which will mature in future periods will be made, so these sums are subtracted from available cash. After all these transactions are finished, there may be a cash balance remaining. This sequence of transactions is represented mathematically as:

$$Ct\text{-}1 + It + \sum_{t=1}^{t-1} (NPtk + IDtk) - \sum_{t=t}^{t+k} NPtk - Ot - Ct = 0.$$

This constraint would be required for each period. For this example, this constraint is represented as a series of six equations, one for each of the periods. Table 2 lists this set of constraints for the sample school district. To complete the linear program formulation, the term structure of interest rates and the initial inflow, outflow, and beginning balance data must be given. In the previous section, it was stated that investments

TABLE 2

The Formulation of a Linear Programming Model For School District

Financial Management

The Objective Function:

Maximize Z = ID11 + ID12 + ID13 + ID14 + ID15 + ID21 + ID22 + ID23 + ID24 + ID31 + ID32 + ID33 + ID41 + ID42 + ID51.

Subject To:

1. The "Cash Balance" Constraints for Each Period -

C0 + I1 - O1 - C1 - NP11 - NP12 - NP13 - NP14 - NP15 = 0.
C1 + I2 - O2 - C2 - NP21 - NP22 - NP23 - NP24 + NP11 + ID11 = 0.
C2 + I3 - O3 - C3 - NP31 - NP32 - NP33 + NP12 + ID12 + NP21 + ID21 = 0.
C3 + I4 - O4 - C4 - NP41 - NP42 + NP13 + ID13 + NP22 + ID22 + NP31 + ID31 = 0.
C4 + I5 - O5 - C5 - NP51 + NP14 + ID14 + NP23 + ID23 + NP32 + ID32 + NP41 + ID41 = 0.
C5 + I6 - O6 - C6 + NP15 + ID15 + NP24 + ID24 + NP33 + ID33 + NP42 + ID42 + NP51 + ID51 = 0.

2. The Term Structure of the Interest Rates on Investment Constraints -

ID11 - .0058NP11 = 0.
ID12 - .0116NP12 = 0.
ID13 - .0225NP13 = 0.
ID14 - .03NP14 = 0.
ID15 - .046NP15 = 0.
ID21 - .0058NP21 = 0.
ID22 - .0116NP22 = 0.
ID23 - .0225NP23 = 0.
ID24 - .03NP24 = 0.
ID31 - .0058NP31 = 0.
ID32 - .0116NP32 = 0.
ID33 - .0225NP33 = 0.
ID41 - .0058NP41 = 0.
ID42 - .0116NP42 = 0.
ID51 - .0058NP51 = 0.

3. The Values of the Monthly Inflows, Outflows, and the Beginning Cash Balance -

C0 = 50000
I1 = 400000
I2 = 250000
I3 = 400000
I4 = 150000
I5 = 100000
I6 = 50000
O1 = 200000
O2 = 200000
O3 = 200000
O4 = 200000
O5 = 200000
O6 = 400000

of 1-2 months duration would earn 7% annual interest or .0058 monthly. An investment of 3-4 months duration would earn 9% annually or .0075 monthly. An investment of 5-6 months duration would earn 11% annual interest or .0092 monthly. Thus ID14 = (.0075)(4)(NP14) = .03NP14. Setting the equation to zero yields IDtk - yk(NPtk) = 0, where y is the monthly interest rate. Table 2 lists the fifteen interest rate formulations for the sample problem. The initial values of the inflows and outflows of each period are also given in Table 2, as well as the beginning cash blance. This completes the formulation of the linear programming example.

The solution of the linear programming formulation can be accomplished through the use of any linear programming computer program generally available at computer centers. For the solution of the example problem, a regular linear programming computer program based on the simplex technique was used. The objective function and constraints as listed in Table 2 were used with the following results.

The optimal solution for the example would be to make the following investments with the resultant interest earned:

NP15 = $250,000	with	ID15 = $11,500.00
NP23 = $ 50,000	with	ID23 = $ 1,125.00
NP31 = $ 98,024	with	ID31 = $ 568.54
NP33 = $101,975	with	ID33 = $ 2,294.45
NP41 = $ 48,593	with	ID41 = $ 281.84

Thus, the total interest earned equals $15,769.83. Based upon the beginning cash balance, the monthly inflows and outflows, and the term structure of interest rates faced by the sample school district, the five investments listed above would yield the greatest amount of interest earnings possible while making sure sufficient cash is available to meet current expenditures.

Discussion of the Linear Programming Financial Planning Model

The linear programming model was developed with the intent to be used as a tool in the decision-making process of school district financial planning. As shown by the hypothetical results, the model produces several types of information:

1. the number of investments that can be made,
2. the timing and amounts of possible investments,
3. the resultant interest earnings of possible investments, and
4. the anticipated cash balances at the end of each period.

These four sets of information are determined in a manner that results in the optimization of the net interest earnings of the school district.

The model is general in nature. The constraints used my be adjusted for inclusion in the model that most closely adheres to the financial environment faced by the school district. The inflows and outflows of the model may be adjusted in a conservative financial package which understates the amount of cash available for investment, or it may be very "tight" resulting in the investment of all surplus cash.

In addition to testing the model on the simple data of the hypothetical school district, the model has been tested in several school districts in Indiana. In all cases, the model indicated that additional investments could have been made which would have resulted in increases in the net interest earnings of the school district.

With fiscal budgets usually in the millions of dollars, school districts have the opportunity to develop substantial revenues through wise financial management. Through the use of this model, as well as other financial decision-making tools available, school district financial planners may better utilize this resource.

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